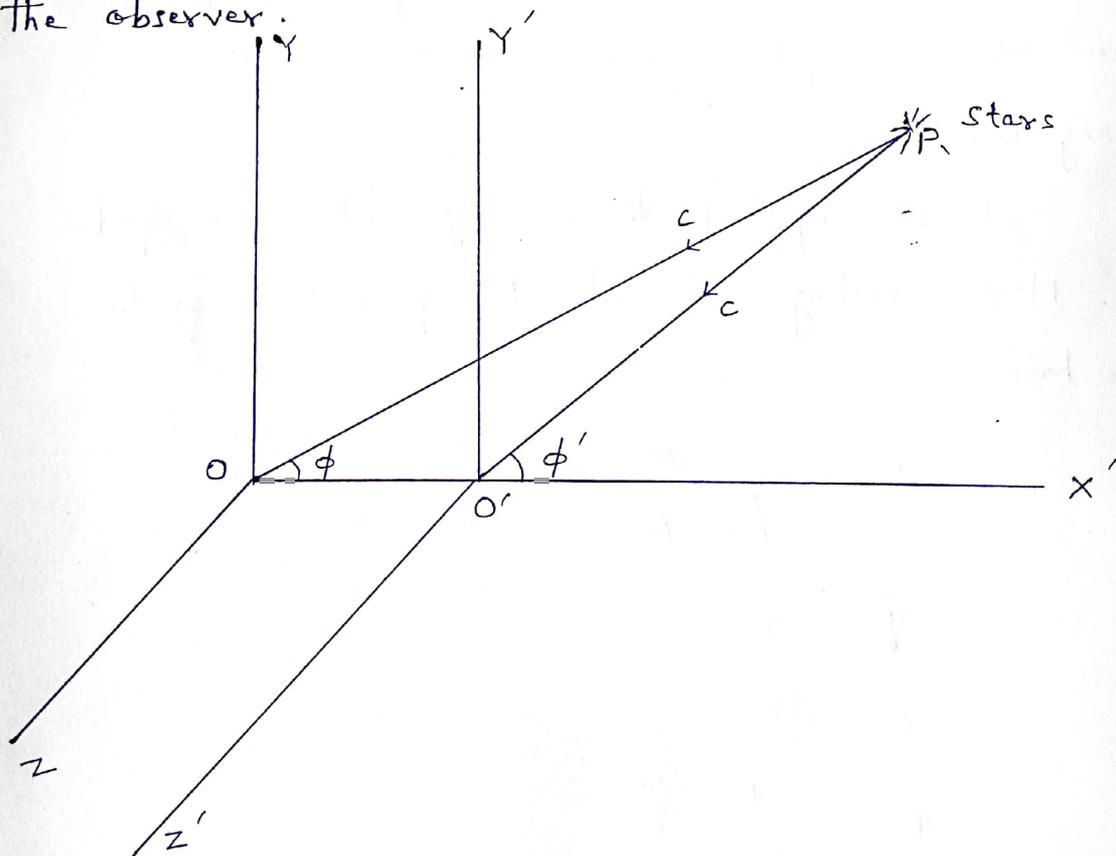


The phenomenon of aberration originally discovered by Bradley in 1727 is very useful to determine the velocity of earth when the velocity of light is known. According to Bradley all the stars except those at the zenith i.e. directly overhead move in elliptical orbit with a period of one year, while those of the zenith move in a circular orbit. Therefore a seasonal change in the position of stars is observed. The phenomenon of aberration results. The speed of light is independent of the medium of transmission, but the direction of light rays depends on the motion of the source emitting light relative to the observer.



As we know that the earth revolves round the sun in the ~~earth~~ orbit, therefore we may

Imagine the sun to the system S while the earth in system S' which is moving with velocity v relative to system S along (+)ve direction of common X -axis. Let the light from a star P be observed by observers O and O' in system S and S' respectively.

Let the angles made by light ray X - Y plane from the star P at any time instant in two systems at O at O' be ϕ and ϕ' respectively. If any particle and P has velocity u in system S and u' in system S' , then we have

$$u = iu_x + ju_y + ku_z$$

$$u' = iu'_x + ju'_y + ku'_z$$

where u_x, u_y and u_z are components of u along three axes in system S and i, j, k are unit vectors along three axes.

Dashes represents the same quantities for system S' . Now according to Lorentz transformation equations, we have

$$x' = \frac{x - vt}{\sqrt{1 - \beta^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \beta^2}}$$

Taking differentials we have

$$dx' = \frac{dx - v dt}{\sqrt{1 - \beta^2}} \quad (1)$$

$$dy' = dy \quad (2)$$

$$dz' = dz \quad (3)$$

$$dt' = \frac{dt - \frac{v dx}{c^2}}{\sqrt{1 - \beta^2}} \quad (4)$$

From eqn (4) and (1) on dividing we get

$$\begin{aligned} u_x' &= \frac{dx'}{dt'} = \frac{dx - v dt}{dt - \frac{v dx}{c^2}} \\ &= \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2} \cdot \frac{dx}{dt}} \end{aligned}$$

$$\text{or } u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} u_x} \quad (5)$$

From (2) and (4)

$$\begin{aligned} u_y' &= \frac{dy'}{dt'} = \frac{dy}{dt - \frac{v}{c^2} \frac{dx}{dt}} \\ &= \frac{\frac{dy}{dt} \sqrt{1 - \beta^2}}{1 - \frac{v}{c^2} \frac{dx}{dt}} \end{aligned}$$

$$= \frac{u_y \sqrt{1 - \beta^2}}{1 - \frac{v}{c^2} u_x} \quad \text{-----} \quad (6)$$

Now from fig. it is clear that the star light travelling in x - y plane with velocity c , has components $c \cos(\pi + \phi)$ along (+)ve direction of X -axis in system S and S' respectively. Also those $+c \sin(\pi + \phi)$ and $+c \sin(\pi + \phi')$ along (+)ve direction of y -axis in system S and S' respectively. Thus we have

$$u_x = +c \cos(\pi + \phi) = -c \cos \phi$$

$$u_y = +c \sin(\pi + \phi) = -c \sin \phi$$

$$u_x' = +c \cos(\pi + \phi') = -c \cos \phi'$$

$$u_y' = +c \sin(\pi + \phi') = -c \sin \phi'$$

$$\frac{u_y'}{u_x'} = \frac{\frac{u_y \sqrt{1 - \beta^2}}{1 - \frac{v}{c^2} u_x}}{u_x - v}$$

$$= \frac{u_y \sqrt{1 - \beta^2}}{u_x - v} \quad \text{-----} \quad (8)$$

Substituting value from (7)

$$\frac{u_y'}{u_x'} = \frac{-c \sin \phi'}{-c \cos \phi'}$$

Continue
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